# MHD Unsteady Memory Convective Flow through Porous Medium with Variable Suction in the presence of Radiation and Permeability

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**ABSTRACT:-** A free convective unsteady visco-elastic flow through porous medium bounded by an infinite vertical porous plate with variable suction, constant heat flux under the influence of transverse uniform magnetic field along with permeability in the presence of radiation has been investigated in the present study. The suction velocity of the porous medium fluctuates with time about the constant mean. Approximate solutions for mean velocity, transient velocity, mean temperature and transient temperature of non-Newtonian flow are obtained. The effects of various parameters such as $P_r$ (Prandtl number), $G_r$ (Grashof number), M (Hartmann number), ω (frequency parameter) and k (permeability parameter) and F (radiation parameter) on the above are depicted.. Expressions for fluctuating parts of velocity '$M_r$' and '$M_i$' are found and plotted graphically and effects of different parameters on them are discussed. Skin friction amplitude and phase are shown graphically and discussed in detail



**Keywords:** Free convection, Walter's liquid B', permeability, radiation and suction.

## INTRODUCTION

The phenomenon of free convection arises in the fluid when temperature and concentration changes cause density variation leading to buoyancy forces acting on the fluid elements.The mass transfer differences effect the rate of heat transfer. In industries, many transport processes exist in which heat and mass transfer take place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion thermo chemical species. The phenomenon of heat and mass transfer frequently exists in chemically processed industries such as food processing and polymer production. Free convection flows are also of great interest in a number of industrial applications such as fiber and granular insulation geothermal system etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Magneto-hydrodynamics is attracting the attention of many authors due to its application in geophysics. In engineering in MHD pumps, MHD bearing etc. at high temperature attained in some engineering devices. Since some fluids can emit and absorb thermal radiation, it is of interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. This is of interest because heat transfer by thermal radiation is becoming of great importance when we are concerned with space application and higher operating temperatures. Soundalgekar and Takhar [1] studied the effects of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly-Vincentine-Gillas equilibrium model. Takhar et.al.[2] also investigated the effect of radiation on MHD free convection flow past a semi-infinite vertical plate for same gas. Muthucumarswamy and Kumar [3] studied the thermal radiation effects on moving infinite vertical plate in presence of variable temperature and mass diffusion.Hussain et.al. [4] studied the effect of radiation on free convection on porous vertical plate.Chamkha el.al.[5] studied the effect of hydro-magnetic combined heat and mass transfer by natural convection from a permeable surface embedded in fluid Saturated porous medium.

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Noushima et al. [6] have studied the hydro magnetic free convective Revlin-Ericksen flow through a porous medium with variable permeability. Suneetha and et al. [7] studied the radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source / sink. Later Vasu et al. [8] studied the radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Prasad et al. [9] have studied the finite difference analysis of radioactive free convection flow past an impulsively started vertical plate with variable heat and mass flux. Seth et al. [10] studied the effects of rotation and magnetic field on unsteady coquette flow in a porous channel. Singh and Kumar [11] have studied the fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip-flow regime. Das et al. [12] have studied the mass transfer effects on unsteady hydro magnetic convective flow past a vertical porous plate in a porous medium with heat source. Reddy and Reddy [13] studied the mass transfer and heat generation effects on MHD free convective flow past an inclined vertical surface in a porous medium.

The aim of this paper is to extend the problem of Maharshi and Tak [14] to memory fluid i.e. Walter's liquid model B′ [17] with variable suction in the presence of radiation and chemical reaction. The mixture of polymethyl mehacrylate and pyridine at $25^0$ C containing 30.5g of polymer per litre behaves very nearly as the above mentioned liquid.

## 2. FORMULATION OF THE PROBLEM

We consider the flow of convective memory fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux under the influence of uniform transverse magnetic field. The $x^*$ - axis is taken along the plate in the upward direction and $y^*$ - axis normal to it. All the fluid properties are assumed to be constant, except that influence of the density variations with temperature is considered only in the body force term. The magnetic field of small intensity $H_0$ is induced in the $y^*$ - direction. Since the fluid is slightly conducting, the magnetic Reynolds number is far lesser than unity; hence the induced magnetic field is neglected in comparison with the applied magnetic field following Sparrow and Cess [15]. The viscous dissipation and Darcy's dissipation terms are neglected for small velocities following Rudraiah et al. [16]. The flow in the medium is entirely due to buoyancy force. The fluid is a grey, absorbing-emitting radiation but non-scattering medium . So under these conditions the flow with variable suction is governed by the following equations:

$$v^* = -v_0(1+\varepsilon e^{i\omega t}) \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = g\beta_1(T^* - T_\infty{}^*) + \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\vartheta u^*}{k^*(t)} - \beta\left(\frac{\partial^3 u^*}{\partial t^*\partial y^{*2}} + v^*\ \frac{\partial^3 u^*}{\partial y^{*3}}\right) - \left(\frac{\sigma\mu^2{}_e H^2{}_0}{\rho}\right)u^* \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k^*}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y^*} \quad (3)$$

The corresponding boundary conditions are

$$\left.\begin{array}{lll} y^* = 0\,, & u^* = 0, & \frac{\partial T^*}{\partial y^*} = \frac{-q}{k}\,, \\ y^* \to \infty\,; & u^* = 0\,, & T^* = T^*_\infty\,, \end{array}\right\} \quad (4)$$

Also

$$-\frac{\partial q_r}{\partial y_*} = 4d\sigma^*\left(T^{*4}_{\ w} - T^{*^4}\right) = 16d\sigma'\,T^{*3}_{\ w}(T^*{}_w - T^*) \quad (5)$$

Introducing the following non-dimensional quantities

$y = \frac{y^* v_0}{\vartheta}$ , $t = \frac{t^* v_0^2}{4\vartheta}$ , $\omega = \frac{4\vartheta\omega^*}{v_0^2}$ , $u = \frac{u^*}{v_0}$ , $G_r = \frac{g\beta_1 q\vartheta^2}{k v_0^4}$ ; $P_r = \frac{\mu c_p}{k}$ ;

$R_m = \frac{\beta v_0^2}{\vartheta^2}$ ; $M = \frac{\sigma\mu_e^2 H_0^2}{v_0^2 \rho}$ ; $\theta = \frac{(T^* - T_\infty^*) k v_0}{q\vartheta}$ ; $F = \frac{16 d\sigma^* {T_w^*}^3 \vartheta^2}{k v_0^2}$ . $k_0 = \frac{k_0^* v_0^2}{\vartheta^2}$ .

The equations (2) – (5) reduces to

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y} = G_r\theta + \frac{\partial^2 u}{\partial y^2} - R_m\left(\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - (1+\varepsilon e^{i\omega t})\frac{\partial^3 u}{\partial y^3}\right) - \left(M + \frac{1}{k_0}\right)u \tag{6}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - F\theta \tag{7}$$

$$\left.\begin{array}{llll} y = 0\,, & u = 0\,, & \frac{\partial\theta}{\partial y} = -1\,, & \\ y \to \infty\,; & u = 0\,, & \theta = 0\,, & , \end{array}\right\} \tag{8}$$

## II. METHOD OF SOLUTION

Let us assume the solution of the form

$$\left.\begin{array}{l} u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + - - - - \\ \theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t}\theta_1(y) + - - - - - \end{array}\right] \tag{9}$$

Substituting equation (9) in equations (6) – (8)

We get

$$R_m u_0''' + u_0'' + u_0' - \left(M + \frac{1}{k_0}\right)u_0 = -G_r\theta_0 \tag{10}$$

$$R_m u_1''' + \left(1 - \frac{iR_m\,\omega}{4}\right)u_1'' + u_1' - \left(M + \frac{1}{k_0} + \frac{i\omega}{4}\right)u_1 = -G_r\theta_1 - u_0' - R_m u_0''' \tag{11}$$

$$\theta_0'' + P_r\theta_0' - P_r F\theta_0 = 0 \tag{12}$$

$$\theta_1'' + P_r\theta_1' - P_r\left(F + \frac{i\omega}{4}\right)\theta_1 = -P_r\theta_0' \tag{13}$$

$$\left.\begin{array}{llll} y = 0\,, & u_0 = 0\,, & \theta_0' = -1, & \\ y \to \infty\,; & u_0 = 0\,, u_1 = 0\,, & \theta_0 = 0\,, & \theta_1 = 0, \end{array}\right\} \tag{14}$$

Equations (10) and (11) are third order differential equations when $R_m \neq 0$ and we have two boundary conditions, so we follow Beard and Walter's [19] and assume $u_0$ and $u_1$ of the form

$$u_0 = u_{01} + R_m u_{02} + 0(R_m^2) \tag{15}$$

$$u_1 = u_{11} + R_m u_{12} + 0(R_m^2) \tag{16}$$

Putting $u_0$ and $u_1$ from (15) and (16) in (10) and (11) and comparing the terms independent of $R_m$ and coefficients of $R_m$ , and neglecting those of $o(R_m^2)$ we get

$$u_{01}'' + u_{01}' - \left(M + \frac{1}{k_0}\right)u_{01} = -G_r\theta_0 \tag{17}$$

$$u_{01}''' + u_{02}'' + u_{02}' - \left(M + \frac{1}{k_0}\right)u_{02} = 0 \tag{18}$$

$$u_{11}'' + u_{11}' - (M + \frac{1}{k_0} + \frac{i\omega}{4})u_{11} = -G_r\theta_1 - u_{01}' \quad (19)$$

$$u_{11}''' - \frac{i\omega}{4}u_{11}'' + u_{12}'' + u_{12}' - (M + \frac{1}{k_0} + \frac{i\omega}{4})u_{12} = -u_{02}' - u_{01}''' \quad (20)$$

With boundary equations

$$\left.\begin{array}{l} y = 0, \quad u_{01} = 0, \quad u_{02} = 0 \quad u_{11} = 0, \quad u_{12} = 0 \\ y \to \infty; \quad u_{01} = 0, \quad u_{02} = 0, \quad u_{11} = 0, \quad u_{12} = 0 \end{array}\right\} \quad (21)$$

The velocity and temperature fields are given by $u = u_0 + \varepsilon u_1 = u_{01} + R_m u_{02} + \varepsilon(u_{11} + R_m u_{12})$ (22)

$$\theta = \theta_0 + \varepsilon\theta_1 \quad (23)$$

Taking real part of solution for the velocity field and temperature field. These can be expressed in terms of fluctuating part as

$$u(y,t) = u_0(y) + \varepsilon(M_r\cos\omega t - M_i\sin\omega t) \quad (24)$$

$$\theta = \theta_0(y) + \varepsilon(N_r\cos\omega t - N_i\sin\omega t) \quad (25)$$

Where

$$M_r + iM_i = u_1(y) \quad (26)$$

$$N_r + iN_i = \theta_1(y) \quad (27)$$

The expressions of transient velocity and transient temperature for $\omega t = \pi / 2$ are given by

$$u\left(y, \frac{\pi}{2\omega}\right) = u_0(y) - \varepsilon M_i \quad (28)$$

$$\theta\left(y, \frac{\pi}{2\omega}\right) = \theta_0(y) - \varepsilon N_i \quad (29)$$

The skin friction at the plate in terms of amplitude and phase is:

$$C_f = \frac{\tau_\omega}{\rho v_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=o} = B_2[(m_1 - m_5) + R_m D_1(m_1 - m_6)] + \varepsilon|\lambda|\cos(\omega t + \alpha) \quad (30)$$

Where

$\lambda_r + i\lambda_i = \lambda$ and $\tan\alpha = \frac{\lambda_i}{\lambda_r}$

## III Result and Discussion

We observe from Figure 1 that for all values considered of Prandtl number Pr, Magnetic number M, Grashof number Gr, Permeability number $k_0$ and Radiation parameter F, the mean transient velocity decreases with decreasing of Gr and increasing of $k_0$ in the presence of radiation parameter F and it further decreases with decreasing of $k_0$ and increasing of M.The mean transient velocity decreases for large values of M and F and increases for small values of M and large values of F.

Figure 2 represents the transient velocity profile for Prandtl number Pr, frequency ω, magnetic number M, Grashof number Gr, permeability $k_0$ and radiation parameter F. The transient velocity increases sharply in the absence of radiation in the vicinity of the plate and then decreases as one move away from the plate but it decreases in the presence of radiation. It is observed that velocity increases with increasing of Gr and F. It is also observed that velocity increases with increasing of frequency ω and $k_0$ but transient velocity decreases with increasing of M and Gr.

Figure 3 represents the mean temperature for Prandtl number Pr and Radiation parameter F and it is observed that mean temperature decreases with increasing of F. Figure 4 represents the effect of Pr and F on the temperature profile for fixed values of $k_0$ ,M,Gr and frequency ω and it is observed that temperature decreases with increasing of F.

Figure 5 and Figure 6 represent the fluctuating parts of transient velocity and it is found that $M_r$ is negative and $M_i$ is positive for all values of Pr, ω ,M ,Gr, $k_0$ and F.It is observed that Mi decreases with increasing of F and it also decreases with increasing of M and frequency ω but increases for increasing of $k_0$ . The value of $M_r$ Increases with decreasing of Gr and it also increases for decreasing of $k_0$ and for increasing of M.

For phase and amplitude of the skin friction we observe from Figure 7 and Figure 8 that phase tanα remains negative for different values of $k_0$ ,Pr and frequency ω .This means that there is always a phase lag. The phase tanα decreases with decreasing of Gr and increasing of M and Pr.The amplitude I λI decreases with decreasing of Gr and increasing of M and further decreases with decreasing of M .It is found that Pr has no impact on I λI .



## IV. Nomenclature

u, v, --- dimensionless velocity components of the fluid; θ ---- dimensionless fluid temperature ;

$u^*, v^*$ -- velocity components of the fluid ; M ---- Hartmann number ;

$T^*$ ---- temperature of the fluid; $T^*_\infty$ ---- temperature of fluid away from the plate ;

$C_p$ --- specific heat at constant pressure; $v_0$ - suction velocity

$x^*, y^*$ -- Co-ordinate axis ; $x, y$ -- dimensionless co-ordinate axis

$T^*_\infty$ -- Temperature of fluid away from the plate; $G_r$ --- Grashof number

$T^*_w$ -- Temperature of fluid at the plate ; , ω ------ the frequency of fluctuation

$R_m$ --- magnetic Reynolds number; $\beta_1$ ---- coefficient of thermal expansion

$k^*$ --- thermal conductivity; $H_0$ ---------- magnetic intensity

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$P_r$ --- Prandtl number;

$\rho$ ----- density;

q ---- heat flux at the plate;

F ---- radiation parameter;

$k_0$ ----- dimensionless permeability parameter;

t --- dimensionless time;

$\vartheta$ ----- kinematic viscosity

$\mu_e$ -------- magnetic permeability

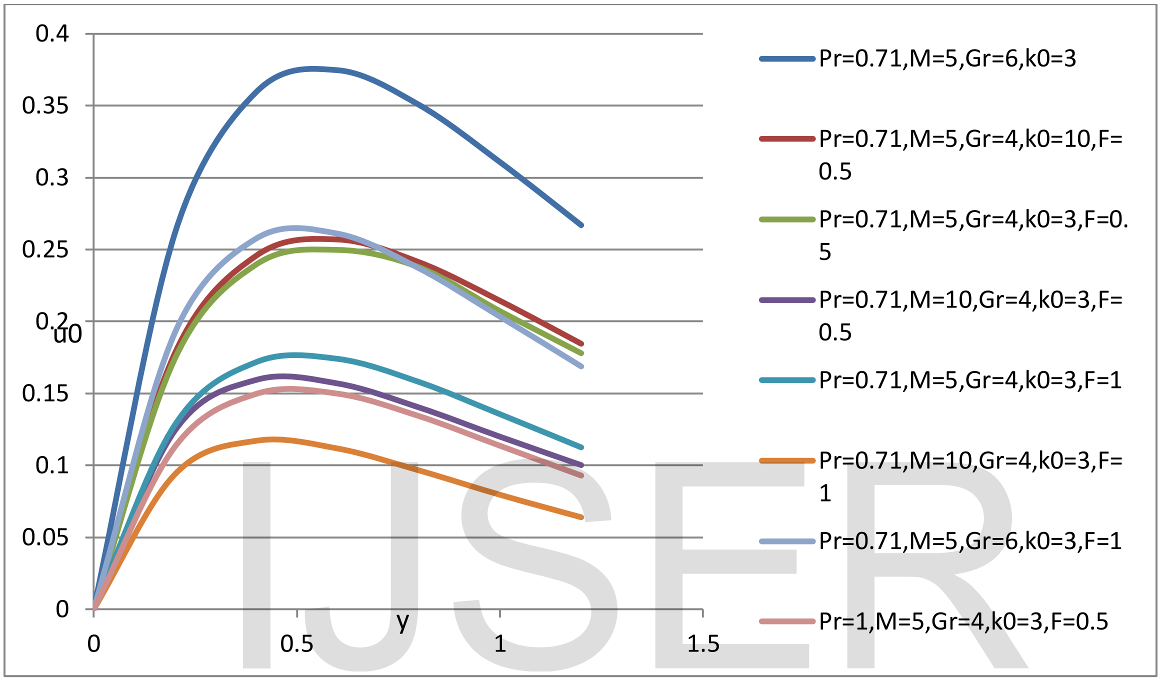
$\sigma$ ------ electrical conductivity

$\beta$ ----- kinematic visco-elasticity,

$t^*$ --- time;

$K^*$ ----- permeability parameter

## V Figures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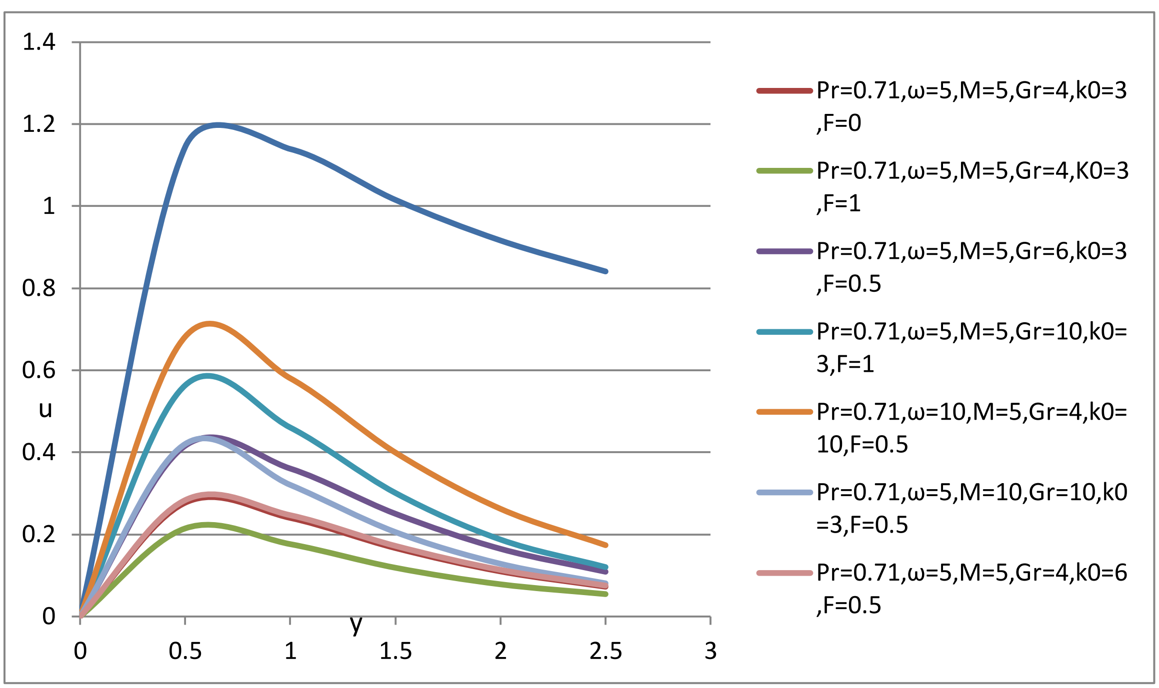
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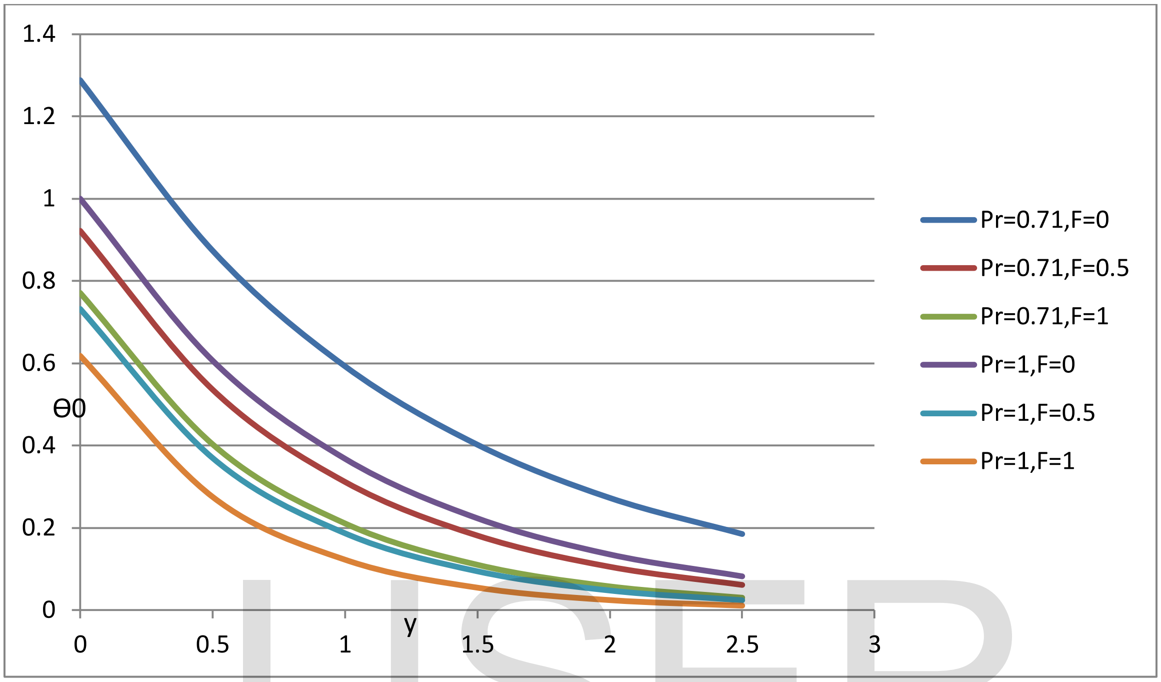
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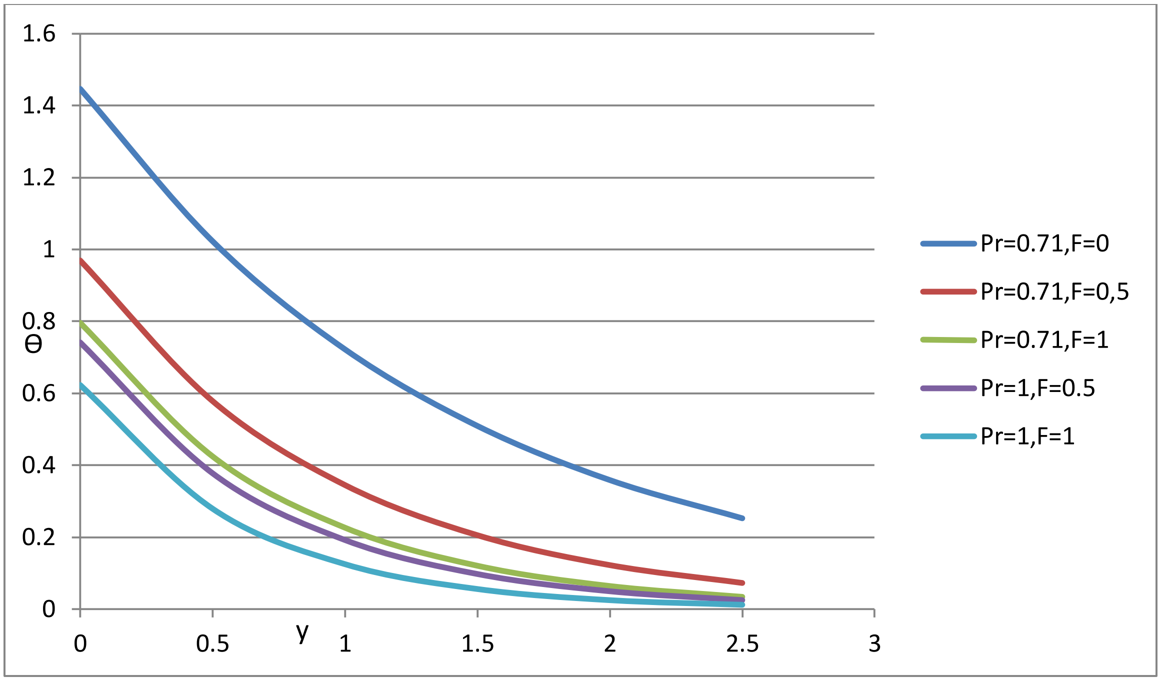

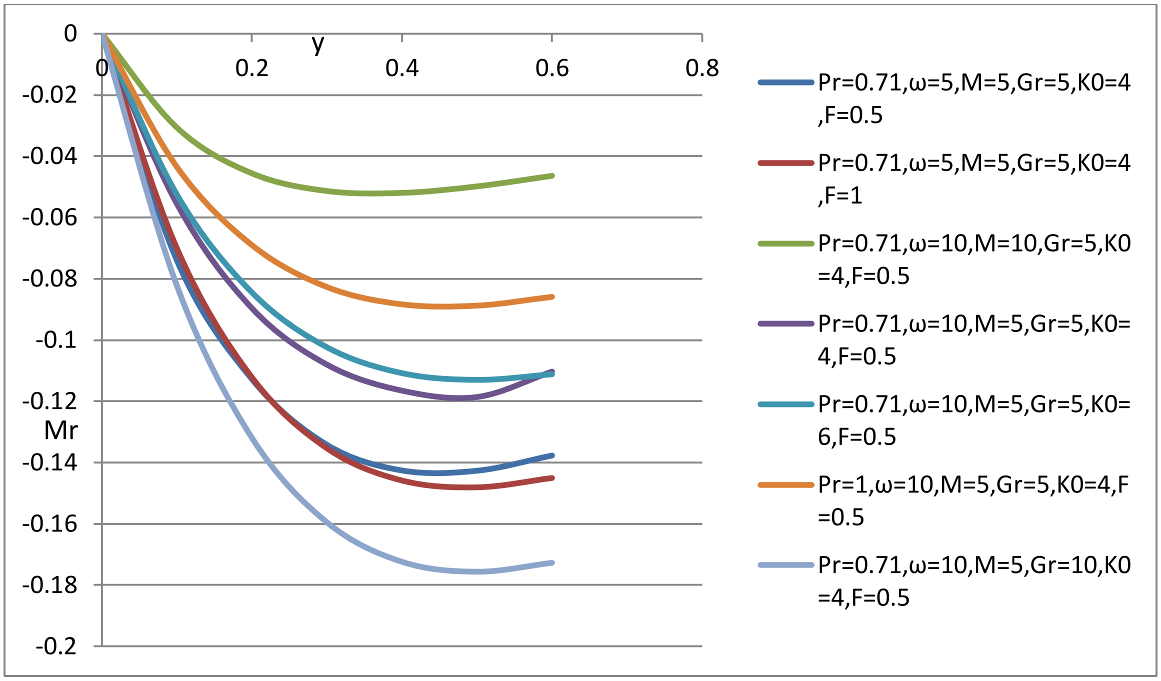
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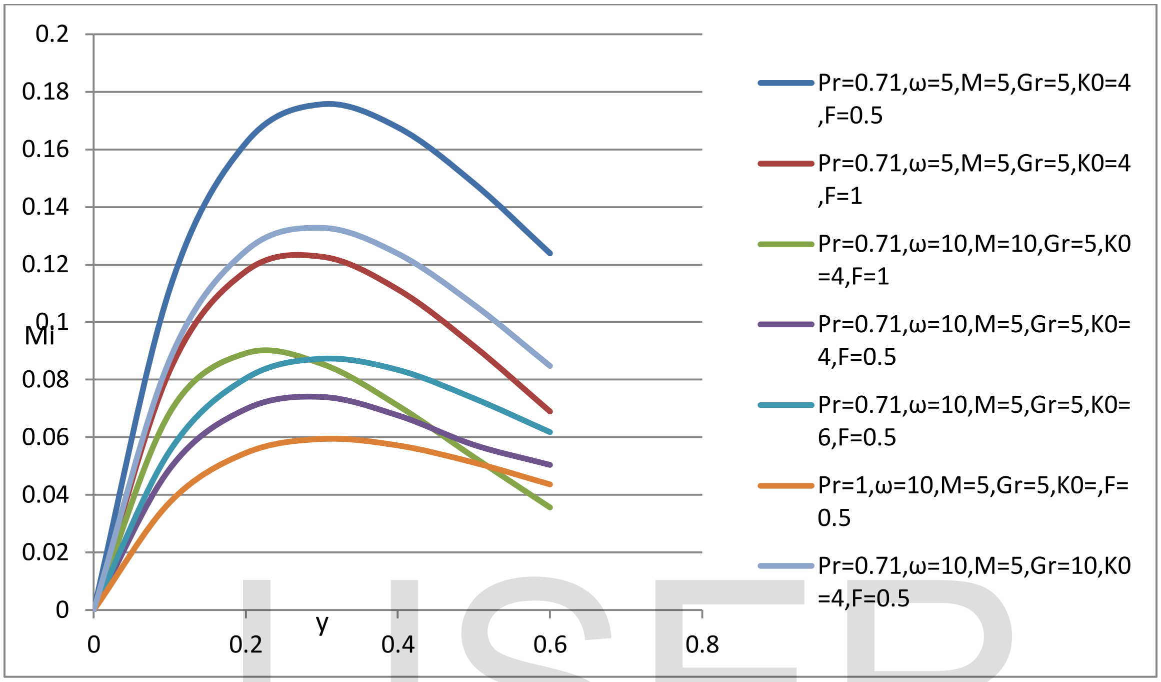

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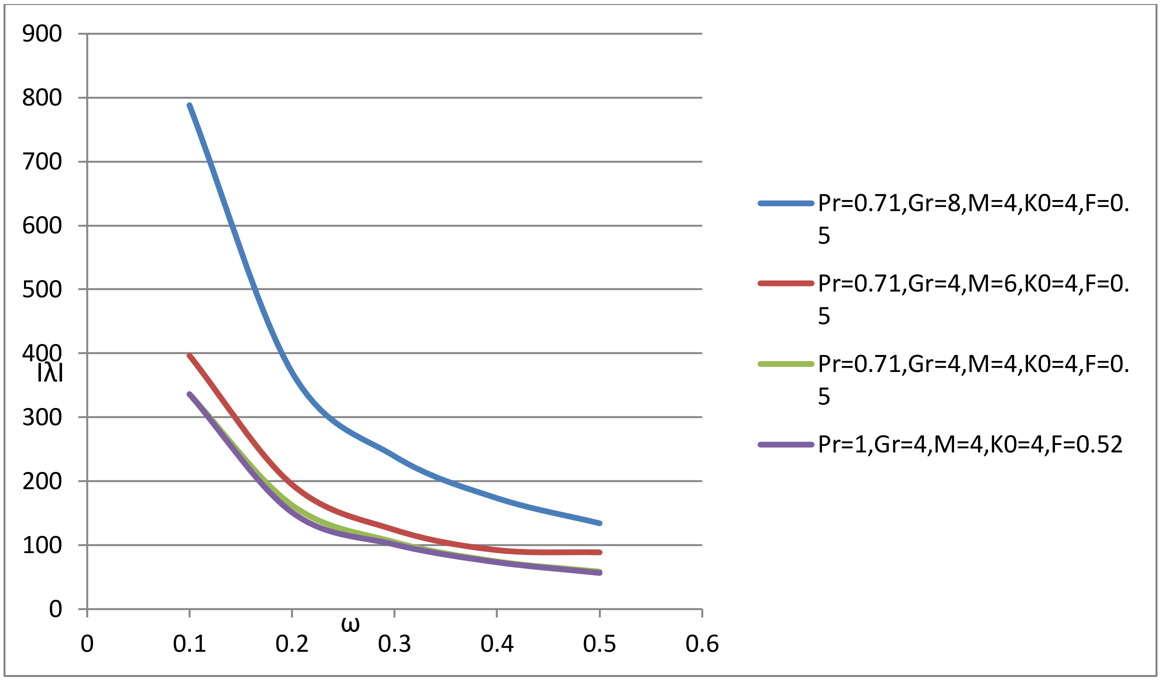

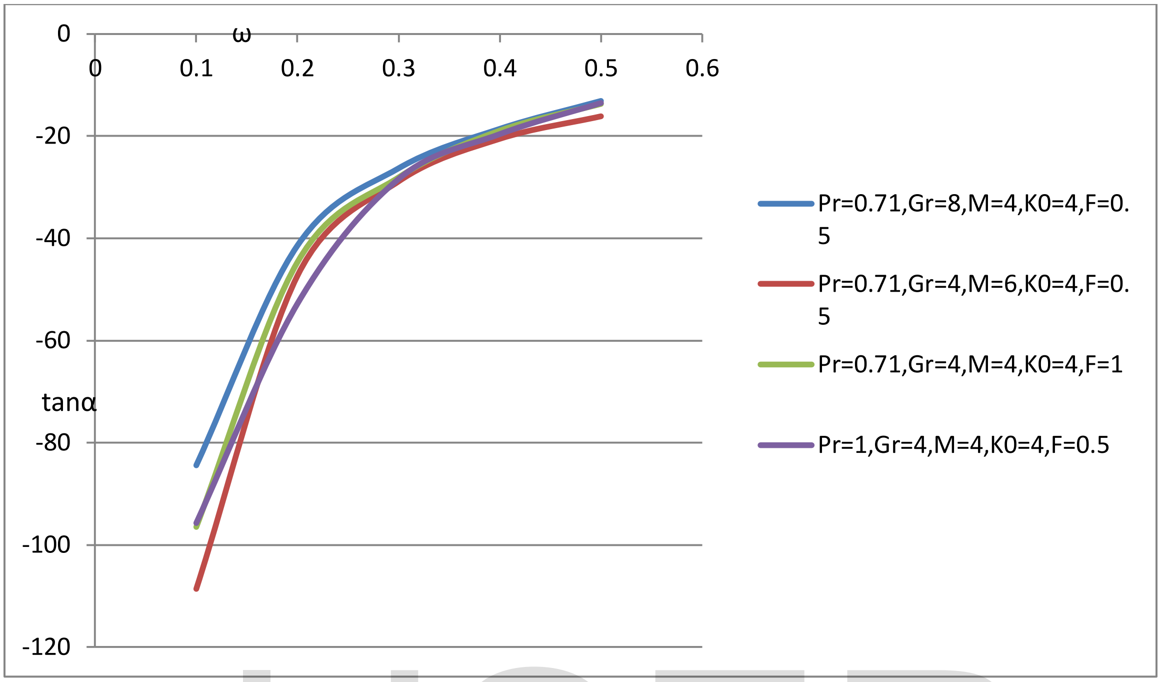
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